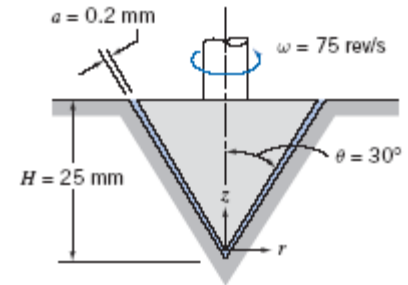


## Problem 2.73

[Difficulty: 4]

**2.73** A viscometer is built from a conical pointed shaft that turns in a conical bearing, as shown. The gap between shaft and bearing is filled with a sample of the test oil. Obtain an algebraic expression for the viscosity  $\mu$  of the oil as a function of viscometer geometry ( $H$ ,  $a$ , and  $\theta$ ), turning speed  $\omega$ , and applied torque  $T$ . For the data given, find by referring to Figure A.2 in Appendix A, the type of oil for which the applied torque is  $0.325 \text{ N} \cdot \text{m}$ . The oil is at  $20^\circ\text{C}$ . *Hint:* First obtain an expression for the shear stress on the surface of the conical shaft as a function of  $z$ .



**Given:** Conical bearing geometry

**Find:** Expression for shear stress; Viscous torque on shaft

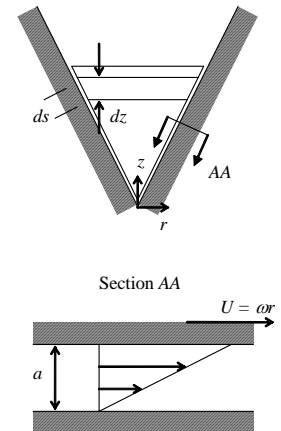
**Solution:**

Basic equation  $\tau = \mu \frac{du}{dy}$   $dT = r \cdot \tau \cdot dA$  Infinitesimal shear torque

Assumptions: Newtonian fluid, linear velocity profile (in narrow clearance gap), no slip condition

$$\tan(\theta) = \frac{r}{z} \quad \text{so} \quad r = z \cdot \tan(\theta)$$

Then 
$$\tau = \mu \frac{du}{dy} = \mu \frac{\Delta u}{\Delta y} = \mu \frac{(\omega \cdot r - 0)}{(a - 0)} = \frac{\mu \cdot \omega \cdot z \cdot \tan(\theta)}{a}$$



As we move up the device, shear stress increases linearly (because rate of shear strain does)

But from the sketch  $dz = ds \cdot \cos(\theta)$   $dA = 2 \cdot \pi \cdot r \cdot ds = 2 \cdot \pi \cdot r \cdot \frac{dz}{\cos(\theta)}$

The viscous torque on the element of area is 
$$dT = r \cdot \tau \cdot dA = r \cdot \frac{\mu \cdot \omega \cdot z \cdot \tan(\theta)}{a} \cdot 2 \cdot \pi \cdot r \cdot \frac{dz}{\cos(\theta)}$$
 
$$dT = \frac{2 \cdot \pi \cdot \mu \cdot \omega \cdot z^3 \cdot \tan(\theta)^3}{a \cdot \cos(\theta)} \cdot dz$$

Integrating and using limits  $z = H$  and  $z = 0$  
$$T = \frac{\pi \cdot \mu \cdot \omega \cdot \tan(\theta)^3 \cdot H^4}{2 \cdot a \cdot \cos(\theta)}$$

Solving for  $\mu$  
$$\mu = \frac{2 \cdot a \cdot \cos(\theta) \cdot T}{\pi \cdot \omega \cdot \tan(\theta)^3 \cdot H^4}$$

Using given data  $H = 25 \cdot \text{mm}$   $\theta = 30 \cdot \text{deg}$   $a = 0.2 \cdot \text{mm}$   $\omega = 75 \cdot \frac{\text{rev}}{\text{s}}$   $T = 0.325 \cdot \text{N} \cdot \text{m}$

$$\mu = \frac{2 \cdot a \cdot \cos(\theta) \cdot T}{\pi \cdot \omega \cdot \tan(\theta)^3 \cdot H^4} \quad \mu = 1.012 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$$

From Fig. A.2, at  $20^\circ\text{C}$ , CASTOR OIL has this viscosity!